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HEAVY HADRONS IN THE RELATIVISTIC QUARK MODEL

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Abstract

Masses of heavy baryons and tetraquarks are calculated in the relativistic quark model using the heavy-quark–light-diquark and diquark-antidiquark approximations, respectively.

1 Introduction

Recently significant experimental progress has been achieved in heavy hadron spectroscopy. Masses of the Ω_c^* , Σ_b , Σ_b^* and Ξ_b baryons as well as masses of several excited charmed baryons have been measured. In the heavy meson sector several new states, such as $X(3872)$, $Y(4260)$, $D_{s0}^*(2317)$, $Z(4430)$ etc., were observed which cannot be simply accommodated in the quark-antiquark ($q\bar{q}$) picture. These states can be considered as indications of the possible existence of exotic multiquark states. In this talk we briefly review our recent

results for the masses of heavy baryons and tetraquarks in the framework of the relativistic quark model based on the quasipotential approach in quantum chromodynamics. We use the heavy-quark–light-diquark and diquark-antidiquark approximations to reduce a very complicated relativistic three- and four-body problem to the subsequent two more simple two-body problems. The first step consists in the calculation of the masses, wave functions and form factors of the diquarks, composed from two light quarks or a light and heavy quark. At the second step, a heavy baryon is treated as a relativistic bound system of a light diquark and heavy quark. The heavy tetraquark is considered to be a bound diquark-antidiquark system. It is important to emphasize that we do not consider a diquark as a point particle but explicitly take into account its structure by calculating the form factor of the diquark-gluon interaction in terms of the diquark wave functions.

2 Relativistic quark model

In the quasipotential approach the two-particle bound state with the mass M and masses of the constituents $m_{1,2}$ in momentum representation is described by the wave function $\Psi(\mathbf{p})$ satisfying the quasipotential equation of the Schrödinger type

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R} \right) \Psi_{d,B,T}(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p}, \mathbf{q}; M) \Psi_{d,B,T}(\mathbf{q}), \quad (1)$$

where

$$\mu_R = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3}, \quad b^2(M) = \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2}.$$

The subscript d refers to the diquark, B refers to the baryon composed of a light diquark and heavy quark, and T refers to the tetraquark composed of a diquark and antidiquark. The explicit expressions for the corresponding quasipotentials $V(\mathbf{p}, \mathbf{q}; M)$ can be found in Refs. ^{1, 2}.

At the first step, we calculate the masses and form factors of the light and heavy diquark. As it is well known, the light quarks are highly relativistic, which makes the v/c expansion inapplicable and thus, a completely relativistic treatment of the light quark dynamics is required. To achieve this goal we closely follow our recent consideration of the spectra of light mesons and adopt the same procedure to make the relativistic potential local by replacing

Table 1: *Masses of the Λ_Q baryons (in MeV).*

$I(J^P)$	Qd state	$Q = c$		$Q = b$		
		M	$M^{\text{exp 3)}$	M	$M^{\text{exp 3)}$	$M^{\text{exp 4)}$
$0(\frac{1}{2}^+)$	$1S$	2297	2286.46(14)	5622	5624(9)	5619.7(2.4)
$0(\frac{1}{2}^-)$	$1P$	2598	2595.4(6)	5930		
$0(\frac{3}{2}^-)$	$1P$	2628	2628.1(6)	5947		
$0(\frac{1}{2}^+)$	$2S$	2772	2766.6(2.4)?	6086		
$0(\frac{3}{2}^+)$	$1D$	2874		6189		
$0(\frac{5}{2}^+)$	$1D$	2883	2882.5(2.2)?	6197		
$0(\frac{1}{2}^-)$	$2P$	3017		6328		
$0(\frac{3}{2}^-)$	$2P$	3034		6337		

$\epsilon_{1,2}(p) = \sqrt{m_{1,2}^2 + \mathbf{p}^2} \rightarrow E_{1,2} = (M^2 - m_{2,1}^2 + m_{1,2}^2)/2M$. Solving numerically the quasipotential equation (1) with the complete relativistic potential, which depends on the diquark mass in a complicated highly nonlinear way¹⁾, we get the diquark masses and wave functions. In order to determine the diquark interaction with the gluon field, which takes into account the diquark structure, we calculate the corresponding matrix element of the quark current between diquark states. Such calculation leads to the emergence of the form factor $F(r)$ entering the vertex of the diquark-gluon interaction¹⁾. This form factor is expressed through the overlap integral of the diquark wave functions.

3 Mass spectra of heavy baryons

We calculated the masses of heavy baryons as the bound states of a heavy quark and light diquark. For the potential of the heavy-quark–light-diquark interaction we used the expansion in p/m_Q ($Q = c, b$). Since the light diquark is not heavy enough for the applicability of a p/m_d expansion, it has been treated fully relativistically. The obtained values of masses of the ground state and excited baryons are given in Tables 1-4 in comparison with available experimental data.

At present the best experimentally studied quantities are the mass spectra of the Λ_Q and Σ_Q baryons, which contain the light scalar or axial vector diquarks, respectively. They are presented in Tables 1, 2. Masses of the

Table 2: *Masses of the Σ_Q baryons (in MeV).*

$I(J^P)$	Qd state	$Q = c$		$Q = b$		
		M	$M^{\text{exp}} \text{ 3, 5)}$	M	$M^{\text{exp}}(\Sigma_b^+)$	$M^{\text{exp}}(\Sigma_b^-) \text{ 6)}$
$1(\frac{1}{2}^+)$	$1S$	2439	2453.76(18)	5805	5807.5(2.5)	5815.2(2.0)
$1(\frac{3}{2}^+)$	$1S$	2518	2518.0(5)	5834	5829.0(2.3)	5836.7(2.5)
$1(\frac{1}{2}^-)$	$1P$	2805		6122		
$1(\frac{1}{2}^-)$	$1P$	2795		6108		
$1(\frac{3}{2}^-)$	$1P$	2799	2802($\frac{4}{7}$)	6106		
$1(\frac{3}{2}^-)$	$1P$	2761	2766.6(2.4)?	6076		
$1(\frac{5}{2}^-)$	$1P$	2790		6083		
$1(\frac{1}{2}^+)$	$2S$	2864		6202		
$1(\frac{3}{2}^+)$	$2S$	2912	2939.8(2.3)?	6222		
$1(\frac{1}{2}^+)$	$1D$	3014		6300		
$1(\frac{3}{2}^+)$	$1D$	3005		6287		
$1(\frac{3}{2}^+)$	$1D$	3010		6291		
$1(\frac{5}{2}^+)$	$1D$	3001		6279		
$1(\frac{5}{2}^+)$	$1D$	2960		6248		
$1(\frac{7}{2}^+)$	$1D$	3015		6262		

ground states are measured both for charmed and bottom Λ_Q , Σ_Q baryons. The masses of the ground state Σ_b and Σ_b^* baryons were first reported very recently by CDF ⁶⁾. CDF also significantly improved the accuracy of the Λ_b mass value ⁴⁾. For charmed baryons the masses of several excited states are also known. It is important to emphasize that the J^P quantum numbers for most excited heavy baryons have not been determined experimentally, but are assigned by PDG on the basis of quark model predictions. For some excited charm baryons such as the $\Lambda_c(2765)$, $\Lambda_c(2880)$ and $\Lambda_c(2940)$ it is even not known if they are excitations of the Λ_c or Σ_c . ¹ Our calculations show that the $\Lambda_c(2765)$ can be either the first radial ($2S$) excitation of the Λ_c with $J^P = \frac{1}{2}^+$ containing the light scalar diquark or the first orbital excitation ($1P$) of the Σ_c with $J^P = \frac{3}{2}^-$ containing the light axial vector diquark. The $\Lambda_c(2880)$ baryon in our model is well described by the second orbital ($1D$) excitation of the Λ_c

¹In Tables 1, 2 we mark with ? the states which interpretation is ambiguous.

Table 3: *Masses of the Ξ_Q baryons with the scalar diquark (in MeV).*

$I(J^P)$	Qd state	$Q = c$			$Q = b$	
		M	$M^{\text{exp 3)}$	$M^{\text{exp 7)}$	M	$M^{\text{exp 8)}$
$\frac{1}{2}(\frac{1}{2}^+)$	$1S$	2481	2471.0(4)		5812	5792.9(3.0)
$\frac{1}{2}(\frac{1}{2}^-)$	$1P$	2801	2791.9(3.3)		6119	
$\frac{1}{2}(\frac{3}{2}^-)$	$1P$	2820	2818.2(2.1)		6130	
$\frac{1}{2}(\frac{1}{2}^+)$	$2S$	2923			6264	
$\frac{1}{2}(\frac{3}{2}^+)$	$1D$	3030			6359	
$\frac{1}{2}(\frac{5}{2}^+)$	$1D$	3042		3054.2(1.3)	6365	
$\frac{1}{2}(\frac{1}{2}^-)$	$2P$	3186			6492	
$\frac{1}{2}(\frac{3}{2}^-)$	$2P$	3199			6494	

with $J^P = \frac{5}{2}^+$ in agreement with the recent spin assignment ⁵⁾ based on the analysis of angular distributions in the decays $\Lambda_c(2880)^+ \rightarrow \Sigma_c(2455)^{0,++}\pi^{+,-}$. Our model suggests that the charmed baryon $\Lambda_c(2940)$, recently discovered by BaBar and confirmed by Belle ⁵⁾, could be the first radial ($2S$) excitation of the Σ_c with $J^P = \frac{3}{2}^+$ which mass is predicted slightly below the experimental value. If this state proves to be an excited Λ_c , for which we have no candidates around 2940 MeV, then it will indicate that excitations inside the diquark should be also considered. ² The $\Sigma_c(2800)$ baryon can be identified in our model with one of the orbital ($1P$) excitations of the Σ_c with $J^P = \frac{1}{2}^-, \frac{3}{2}^-$ or $\frac{5}{2}^-$ which predicted mass differences are less than 15 MeV. Thus masses of all these states are compatible with the experimental value within errors.

Mass spectra of the Ξ_Q baryons with the scalar and axial vector light (qs) diquarks are given in Tables 3, 4. Experimental data here are available mostly for charm-strange baryons. We can identify the $\Xi_c(2790)$ and $\Xi_c(2815)$ with the first orbital ($1P$) excitations of the Ξ_c with $J^P = \frac{1}{2}^-$ and $J^P = \frac{3}{2}^-$, respectively, containing the light scalar diquark, which is in agreement with the PDG ³⁾ assignment. Recently Belle ⁹⁾ reported the first observation of two baryons $\Xi_{cx}(2980)$ and $\Xi_{cx}(3077)$, which existence was also confirmed by BaBar ⁷⁾. The $\Xi_{cx}(2980)$ can be interpreted in our model as the first radial

²The Λ_c baryon with the first orbital excitation of the diquark is expected to have a mass in this region.

Table 4: *Masses of the Ξ_Q baryons with the axial vector diquark (in MeV).*

$I(J^P)$	Qd state	$Q = c$				$Q = b$
		M	$M^{\text{exp}} 3)$	$M^{\text{exp}} 9)$	$M^{\text{exp}} 7)$	M
$\frac{1}{2}(\frac{1}{2}^+)$	1S	2578	2578.0(2.9)			5937
$\frac{1}{2}(\frac{3}{2}^+)$	1S	2654	2646.1(1.2)			5963
$\frac{1}{2}(\frac{1}{2}^-)$	1P	2934				6249
$\frac{1}{2}(\frac{1}{2}^-)$	1P	2928				6238
$\frac{1}{2}(\frac{3}{2}^-)$	1P	2931				6237
$\frac{1}{2}(\frac{3}{2}^-)$	1P	2900				6212
$\frac{1}{2}(\frac{5}{2}^-)$	1P	2921				6218
$\frac{1}{2}(\frac{1}{2}^+)$	2S	2984		2978.5(4.1)	2967.1(2.9)	6327
$\frac{1}{2}(\frac{3}{2}^+)$	2S	3035				6341
$\frac{1}{2}(\frac{1}{2}^+)$	1D	3132				6420
$\frac{1}{2}(\frac{3}{2}^+)$	1D	3127				6410
$\frac{1}{2}(\frac{3}{2}^+)$	1D	3131				6412
$\frac{1}{2}(\frac{5}{2}^+)$	1D	3123			3122.9(1.3)	6403
$\frac{1}{2}(\frac{5}{2}^+)$	1D	3087		3082.8(3.3)	3076.4(1.0)	6377
$\frac{1}{2}(\frac{7}{2}^+)$	1D	3136				6390

(2S) excitation of the Ξ_c with $J^P = \frac{1}{2}^+$ containing the light axial vector diquark. On the other hand the $\Xi_{cx}(3077)$ corresponds to the second orbital (1D) excitation in this system with $J^P = \frac{5}{2}^+$. The new charmed baryons $\Xi_c(3055)$ and $\Xi_c(3123)$, very recently announced by BaBar¹⁰⁾ can be interpreted in our model as the second orbital (1D) excitations of the Ξ_c with $J^P = \frac{5}{2}^+$ containing scalar and axial vector diquarks, respectively. Few months ago the D0 Collaboration reported the discovery of the Ξ_b^- baryon. The CDF Collaboration⁸⁾ confirmed this observation and gave the more precise value of its mass. Our model prediction is in a reasonable agreement with these new data.

4 Masses of heavy tetraquarks

To calculate the masses of heavy tetraquarks we considered them as the bound states of a heavy diquark and antidiquark. In Table 5 we compare our results (EFG²⁾) for the charm diquark-antidiquark bound states with the predictions

Table 5: *Comparison of theoretical predictions for the masses of charm diquark-antidiquark states $cq\bar{c}\bar{q}$ (in MeV) and possible experimental candidates.*

State J^{PC}	Theory			Experiment	
	EFG	Maiani et al.	Maiani et al. ($c\bar{s}\bar{c}\bar{s}$)	state	mass
$1S$					
0^{++}	3812	3723			
1^{++}	3871	3872 [†]		$X(3872)$	3871.9(0.5)
1^{+-}	3871	3754			
0^{++}	3852	3832			
1^{+-}	3890	3882			
2^{++}	3968	3952		$Y(3943)$	3943(11)(13)
$1P$					
1^{--}	4244		4330(70)	$Y(4260)$	4259(8)($\frac{2}{6}$)

[†] input

of Ref. ¹¹⁾. The differences in some of the mass values can be attributed to the substantial distinctions in the used approaches. We describe the diquarks dynamically as quark-quark bound systems and calculate their masses and form factors, while in Ref. ¹¹⁾ they are treated only phenomenologically. Then we consider the tetraquark as purely the diquark-antidiquark bound system. In distinction Maini et al. consider a hyperfine interaction between all quarks which, e.g., causes the splitting of 1^{++} and 1^{+-} states arising from the SA diquark-antidiquark compositions. From Table 5, where we also give possible experimental candidates for the neutral tetraquarks with hidden charm, we see that our calculation supports the assumption ¹¹⁾ that $X(3872)$ can be the axial vector 1^{++} tetraquark state composed from the scalar and axial vector diquark and antidiquark in the relative S state. On the other hand, in our model the lightest scalar 0^{++} tetraquark is predicted to be above the open charm threshold $D\bar{D}$ and thus to be broad, while in the model ¹¹⁾ it lies few MeV below this threshold, and thus is predicted to be narrow. Our 2^{++} tetraquark also lies higher than the one in Ref. ¹¹⁾. We find that $Y(4260)$ cannot be interpreted as P state 1^{--} of charm-strange diquark-antidiquark, since its mass is found to be ~ 200 MeV higher. A more natural tetraquark interpretation could be the P state ($[cq]_{S=0}[\bar{c}\bar{q}]_{S=0}$) which mass is predicted in our model to be close to the mass of $Y(4260)$ (see Table 5). Then the $Y(4260)$ would decay dominantly into $D\bar{D}$ pairs.

5 Conclusions

We found that presently available experimental data for the masses of the ground and excited states of heavy baryons can be accommodated in the picture treating a heavy baryon as the bound system of the light diquark and heavy quark, experiencing orbital and radial excitations between these constituents. It was argued that the $X(3872)$ and $Y(4260)$ can be the neutral charm tetraquark states. If they are really tetraquarks, one more neutral and two charged tetraquark states should exist with close masses.

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References

1. D. Ebert, R. N. Faustov and V. O. Galkin, Phys. Rev. D **72**, 034026 (2005); arXiv:0705.2957 [hep-ph].
2. D. Ebert *et al.*, Phys. Lett. B **634**, 214 (2006); arXiv:0706.3853 [hep-ph].
3. Particle Data Group, , W.-M. Yao *et al.*, J. Phys. G **33**, 1 (2006).
4. D. Acosta *et al.* [CDF Collaboration], Phys. Rev. Lett. **96**, 202001 (2006).
5. B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **98**, 012001 (2007); R. Mizuk *et al.* [BELLE Collaboration], arXiv:hep-ex/0608043.
6. T. Aaltonen *et al.* [CDF Collaboration], Phys. Rev. Lett. **99**, 202001 (2007).
7. B. Aubert *et al.* [BABAR Collaboration], arXiv:hep-ex/0607042.
8. V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. Lett. **99**, 052001 (2007); T. Aaltonen *et al.* [CDF Collaboration], Phys. Rev. Lett. **99**, 052002 (2007).
9. R. Chistov *et al.* [BELLE Collaboration], Phys. Rev. Lett. **97**, 162001 (2006).
10. B. Aubert *et al.*, arXiv:0710.5763 [hep-ex].
11. L. Maiani, F. Piccinini, A. D. Polosa and V. Riquer, Phys. Rev. D **71**, 014028 (2005); Phys. Rev. D **72**, 031502(R) (2005).